Transition to Advanced Mathematics

Problem-a-day Study Guide

Friday

Prove the following statement form is a tautology using a truth table.

$$\big((P \Rightarrow Q) \wedge (R \Rightarrow S) \wedge (P \vee R)\big) \Rightarrow (Q \vee S)$$

Saturday

Prove the following statement

$$\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(xy + y = 7)$$

Sunday

Prove that $\sqrt{17}$ is irrational.

Monday

Prove that $(A \times B) \cap (B \times A) \subseteq (A \cap B) \times (A \cap B)$

Tuesday

Let I be an arbitrary index set and A_i sets indexed by I. Prove or disprove:

$$\left(\bigcup_{i\in I} A_i\right) - B = \bigcup_{i\in I}(A_i - B)$$

Wednesday

Use induction to prove that:

$$\sum_{m=1}^{n} \frac{1}{(2m-1)(2m+1)} = \frac{n}{2n+1}$$

Thursday

Define a relation R on $\mathbb{Z}^2$ via $(a, b)R(x, y)$ if and only if $a \equiv_4 x$ and $b \equiv_5 y$. Prove or disprove that R is an equivalence relation.

Friday

Solve $17x^2 + 4 \equiv 32 \bmod 50$

Saturday

Show that the function f, below, is one-to-one.

$$f\colon \mathbb{R} \to \mathbb{R}^2$$
$$x \to (x^2, x^3)$$

Sunday

The function f, below, is not invertible. Define the largest possible restriction, $g \coloneqq f|_S$ such that g is invertible. Then find the rule that defines g.